18. Forwards and Futures

This is the first of a series of three lectures intended to bring the money view into contact with the finance view of the world. We are going to talk first about interest rate forwards and futures, then interest rate swaps, then credit default swaps. I have been treating forwards as a kind of swap from the beginning, (as all banking is swap of IOUs), but even so it will take a little bit of doing to connect this up with the finance view of swaps. Here is an idea to have in mind to keep you oriented, the idea of "Banking as Advance Clearing".

Remember how we began the course, in lectures 5-9 on "Banking as a Clearing System". In those lectures, we saw how, at any moment, a particular intertemporal pattern of cash flows and cash commitments resolves itself into a particular pattern of clearing and settlement at a moment in time. Deficit agents buy cash (borrow) today to delay settlement, and the elastic availability of loans is the essential source of elasticity in the payments system. By means of **credit**, <u>current</u> imbalances in the pattern of cash inflows and outflows are pushed into the future where, hopefully, they can be offset against a pattern of imbalances going the other way. The cost of pushing those imbalances into the future is the current money rate of interest, which operates therefore as a symptom of the degree of imbalance but also as an incentive to pay up soon; here is the discipline in the system. Financial crisis arises when delaying tactics no longer work; it is an extreme form of discipline.

What I mean now about advance clearing is the way that emerging imbalances <u>in the future</u> show up as cash flow imbalances in the present, again with the money rate of interest serving as a symptom, and discipline. In finance, the future determines the present, but no one knows the future, so there can be multiple views of what the future will look like. How does it happen that one path gets chosen over other possibilities; how does it happen meanwhile that diverse views get coordinated? One way is by market pricing of different views, and by the effect of that pricing on behavior that operates through the survival constraint.

At one extreme, if the market changes its mind about your view of the future, you may have difficulty rolling your funding. The current survival constraint is thus a key mechanism through which one future path gets chosen over all the others. But there are subtler paths at work as well, through which ideas about the future cause changes in cash flows today, which make the survival constraint looser for some people and tighter for others. Today, we explore one of them, namely the cash flow consequences of changes in futures prices.

Forwards and Futures

Suppose a firm has ordered a machine for delivery three months from now. The firm plans to pay for the machine by borrowing, but is concerned that interest rates three months from now might be higher. The firm can lock in a borrowing rate by engaging in a forward contract with a bank. We can think of that forward contract as a swap of IOUs, as follows:

Firm A			Bank	
Assets	Liabilities		Assets	Liabilities
3 month deposit	6 month loan		6 month loan	3 month deposit

We have seen in a previous lecture that the swap of IOUs will have zero present value if the deposit and the loan both pay the forward rate of interest F[3,6] defined by forward interest parity

$$(1+R[0,3])(1+F[3,6]) = (1+R[0,6])$$

We have also seen that, because of the failure of the expectations hypothesis, in general

$$F[3,6] > E_0R[3,6]$$

This empirical regularity provides incentive for the bank to enter into the forward contract. The forward loan is more profitable on average than the spot loan.

This swap of IOUs solves the problem of the firm, but creates a problem for the bank, since in three months the bank will have to come up with the money to lend to the firm, and at that moment it is entirely possible that R[3,6] will be greater than F[3,6], so leaving the bank with a loss. Ideally the bank (or the banking system as a whole) has another client with exactly offsetting needs, i.e. a firm that wants to lock in a lending rate by engaging in a forward contract as follows.

Firm B			Bank	
Assets	Liabilities		Assets	Liabilities
6 month deposit	3 month loan		3 month loan	6 month deposit

You can see how these two contracts exactly offset each other on the balance sheet of the bank. The bank is borrowing and lending for 3 months, and borrowing and lending for 6 months. So the bank has no net exposure. The important point to notice is that this combination of forward contracts have essentially <u>cleared today</u> a future payment from Firm B (the ultimate lender) to Firm A (the ultimate borrower). That's the sense in which the forward market can be considered to be about advance clearing. There will be cash flows in three months between surplus and deficit firms, but they are all pre-arranged today.

Now in general there is no reason to expect that forward contracts all net out in this way on the balance sheet of any single bank. Even when banks trade their forward exposure with one another (using FRAs) there is no reason to expect that forward contracts all net out in the banking system as a whole. That means that the banking system will be left with a net exposure to the risk that the future spot rate of interest will be higher than the current forward rate. Banks will not hold this risk unless they are compensated by an expectation of profit. The source of this profit is movement in the forward rate away from expected spot.

As we have seen repeatedly in this course, this is one possible explanation for the empirical failure of the expectations theory of the term structure. The difference between the current forward rate and the current expectation of the future spot rate is just the expected profit from an unhedged forward exposure. The point is that the imbalance between future cash flows and cash commitments shows up as distortion of the current forward interest rate away from the expected future spot interest rate.

There is more to it than this. If the forward imbalance is large, then the current price distortion will be large, and that means that the expected profit from an unhedged forward exposure will be large. This expected profit can be expected to attract speculators in the larger economy, outside the banking system, to hold the exposure that the banks cannot or are unwilling to bear. Conceptually we will think of the **futures** market as the place where the banking system sells off its excess **forward** exposure to speculators in the outside economy. Futures are forwards that are marked to market daily, with any changes in value settled daily. Distortions that affect forward rates will also affect futures rates, and hence current cash flows..

Chain of Hedges

Client	Bank	Banking System	Futures Exch.	
F[3,6]	Forward contract	FRA	Futures	ER[3,6]

We know from the failure of EH that forward interest rates tend to be upward biased forecasts of future spot rates. The bias is often thought of as a kind of liquidity premium, but people have had a hard time explaining just what risk that premium is compensating for. More generally we observe the following pattern

Forward rate > futures rate > expected spot

This pattern gives everyone a profit incentive to enter into the trade. To understand these effects, we now backtrack and build up a somewhat more formal account of forwards and futures, now using the more typical language of finance.

Forwards

Start with forwards. Forward contracts are promises to deliver goods at future time T at a given price K. The classic example is that of the wheat farmer who has a natural long position in wheat and the baker who has a natural short position. Both face price risk. If they can arrange a forward contract, however, they can lock in the future price of wheat, and both can be made better off. We say they each **hedge** their natural forward exposure by taking an opposite position in a formal forward contract.

When the time comes to settle, the spot price of wheat is likely to be different from the contracted delivery price. In this sense one side "wins" ex post. In fact, we can track these "winnings" over the life of the contract as the value of the contract changes. But in forward contracts these winnings are only notional. No matter what happens to the spot price, at delivery date the short delivers the contracted good to the long, and the long delivers the contracted price to the short in money.

For our purpose we want to think about the case where the underlying is not a physical commodity like wheat but a financial instrument like a Treasury bond. (Or a bank time deposit, such as a Eurodollar deposit.) It's easiest to think about the case where the underlying is a zero coupon riskless bond that yields no cash income and has no carrying cost. We can write our Forward Interest Parity condition in price terms as follows:

$[1/1+F(3,6)] = [1/1+R(0,6)][1+R(0,3)]$

where the first term is the forward delivery price, the second is the current spot price, and the third the interest rate between now and the forward date.

Now think about how the forward price changes over time. The equation above is the forward rate at time zero. At time 1, 2, 3 we have the following

$[1/1+F_1(3,6)] = [1/1+R(1,6)][1+R(1,3)]$

$[1/1+F_2(3,6)] = [1/1+R(2,6)][1+R(2,3)]$

$[1/1+F_3(3,6)] = [1/1+R(3,6)]$

There is no reason at all to expect that these forward rates are the same as the period zero forward rate. That means that the forward contract established at time zero will change in value throughout time. To help us think about that change, and to connect this discussion up with standard finance treatments, it will be useful to recast the discussion in continuous time by introducing some new notation:

At time 0 when the contract is written, we have the following formula relating the forward delivery price K to the current spot price S_0:

[1] $$K = S_0e^{rT}.$$ (See equation 3.5 in Hull 5th ed.)

Don't let this equation scare you. It is nothing more than a version of our familiar forward interest parity condition. Think of the forward price K as 1/(1+F[3,6]), the spot price S_0 as 1/(1+R[0,6]) and the interest rate term e^{rT} as (1+R[0,3]).

To see how this is an arbitrage condition, think about how you would make money if the condition does not hold:

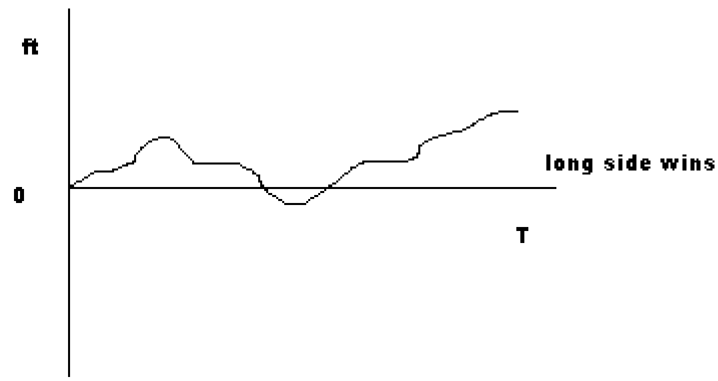
$K > S_0e^{rT}$, buy the bond spot and sell it forward. In this way you lock in a rate of return greater than the rate of interest r at which you can borrow money to finance the trade.

$K < S_0e^{rT}$, sell the bond spot and buy it forward. In this way you lock in a borrowing rate lower than the rate of interest r at which you can lend the money you receive from selling the bond.

The important point to emphasize is that, for forward contracts, the delivery price K is fixed for the life of the contract. Hence, over the life of the contract, the value of the forward contract will change,

[2] $$f_t = S_t - Ke^{-r(T-t)},$$ for 0<t<T. (See equation 3.9 in Hull)

Note that there is a time subscript on both f and S, but K is fixed. At t=0, we have $S_0 = Ke^{-rT}$, so $f_0=0$ when the forward contract is signed. At t=T, we have $f_T = S_T - K$. This is the notional winning we talked about above. In between time 0 and time T, the value of the forward contract fluctuates, depending mainly on the fluctuating spot price of the underlying zero coupon bond.



In most forward contracts, at the final date the long side pays the short side the agreed price K and receives the agreed underlying, which is worth S_T. In interest rate forward contracts however, "cash settlement" is the rule. Instead of delivering the bond for K, the short side delivers the current spot price of the bond in return for the payment K. This means net cash payment of the final value $f_T=S_T-K$ from short to long if positive and from long to short if negative. In cash settlement, the notional winnings become real cash flows at time T.

Futures

A futures contract is like a forward except that all changes in the value of the contract f_t are instead absorbed in changes in the delivery price, which is therefore called the futures price, F_t. F_t is reset every day so that f_t is zero. In other words, the **futures price is that price at which the analogous forward contract has a current value of zero**.

$$0 = S_t - F_t e^{-r(T-t)}$$

[3] $$F_t = S_t e^{r(T-t)}$$

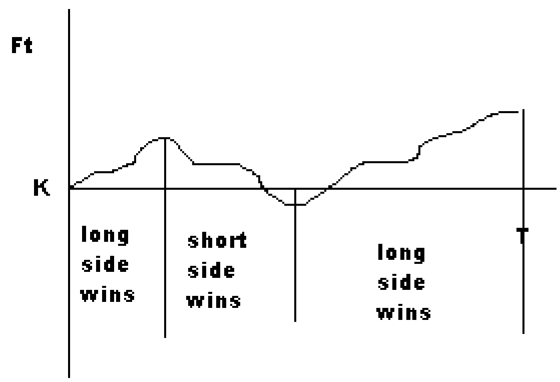
This equation is sometimes called **future-spot parity**.[1] Notice that at origination the futures price is equal to the contract price in a forward contract. (Compare equation [3] and equation [1]). And at expiry the futures price is equal to the spot price. In algebra this means that at t=0, F_t=K, and at t=T, $F_T=S_T$.

The big difference between forwards and futures is that the daily "winnings" that come from changing spot prices are not at all notional. In fact they are actually paid out daily over the life of the contract. In a forward contract, the only payment flow is at the end, and the amount of that payment is fixed by the contract from the very beginning. In a futures contract, payments

[1] I am abstracting from interest paid or received on the fluctuating balances as the futures price changes. That is one reason my formula may be a bit simpler than the formula you may have learned in a previous finance course.

are being made all along the life of the contract, whenever the futures price changes. This is called "mark to market".

Concretely, these payments involve additions and subtractions from "margin accounts" held at the futures clearinghouse. It is significant that both the long and short side have to put up margin, because at the moment the contract is entered, both are in a sense equally likely to lose and so equally likely to have to make a payment to the other side. You can think of these margin accounts as similar to bank deposits, but in fact the clearinghouse will accept securities for the purpose. They have to be liquid securities however, and at the end of the day the securities are repriced to reflect any change during the day. Thus the collateral underlying the futures contract as well as the futures contract itself are both marked to market every day.

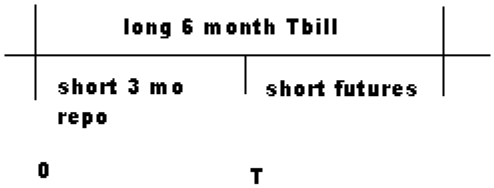


The cumulative payment on the futures is the same as the final payment on the forward, but for the futures the cash flows come about every day during the life of the contract. This is a very concrete way in which views about the future are settled today.

Monetary Issues

Stigum (718-722) talks about a trade involving spot 6 month bills, the 3 month ahead futures contract on the bills, and the 3 month riskfree repo rate. It starts with buying the 6 month bill for spot price S_0 using money borrowed at the repo rate r. Borrowing short term in order to lend long term however exposes you to price risk, since you don't know the rate at which you will be able to finance the second three months of the bill. To hedge that price risk, you short 3 month Treasury bill futures at the futures price F_t. Then, whatever happens to the Tbill price is exactly countered by whatever happens to the futures price. In this way you hedge all price risk. The question then is, given that you have hedged all price risk, why would you ever expect to profit from this trade, and hence why would anyone do it?

Here is a diagram showing the trade in question:



One way to understand this trade is that the trader is long a forward contract (the combo of the Tbill and repo) and short the corresponding futures contract. This way of putting the matter makes it even more puzzling why the trade would ever make a profit. To see why such a trade might be profitable, let's return to the relationship between futures and current spot prices that we talked about previously, under the name futures-spot parity. The same relationship Stigum calls "full carry pricing". Deviations from full carry pricing offer opportunities for arbitrage profit, as below:

$F_t = S_t e^{r(T-t)}$ full carry pricing, no arbitrage profit

$F_t > S_t e^{r(T-t)}$ cash and carry arbitrage:

1) Short futures, long underlying at S_0, finance by borrowing at r
2) At futures expiry, deliver underlying for spot S_T, repay loan

$F_t < S_t e^{r(T-t)}$ reverse cash and carry arbitrage:

1) Long futures, short underlying at S_0, invest proceeds at r
2) At futures expiry, pay spot S_T for underlying

Stigum uses a somewhat different language when she talks about the difference between an "implied repo rate" and the actual repo rate, but she is talking about exactly the same thing. We understand what the actual repo rate is—it is the rate paid on the short term repo (the "carry" part of the trade) that is used to purchase the Treasury bill (the "cash" part of the trade). The implied repo rate is the short term return that is locked in by the combination of the cash bill and the short futures position. Observe that we know the price at which we can buy the bill, and we know the price at which we can sell the bill (i.e. the futures price).

Define the implied repo rate by the equation

$$F_t = S_t e^{\rho(T-t)}$$

The implied repo rate ρ is thus the borrowing rate that would have to hold in order for futures prices to satisfy full carry pricing. Now we can express the arbitrage opportunities as a deviation of the implied repo rate from the actual repo rate:

$\rho = r$	full carry pricing, no arbitrage profit
$\rho > r$	cash and carry arbitrage
$\rho < r$	reverse cash and carry arbitrage

The arbitrage profit in the cash and carry trade arises from the fact that you can borrow at a lower rate than you can lend. Put that way, it is astonishing that such a relationship would ever hold for more than an instant. Why doesn't everyone do it, and in volume sufficient to eliminate the arb?

The cash and carry arbitrage is long forward and short futures. What is the risk in that position that might command a premium for bearing it? If the forward rate is typically greater than the expected spot, that means we can expect to gain by borrowing short and lending long. Our long forward interest rate position should be increasing in value. But at the same time our short futures interest rate position should be decreasing in value. These two positions more or less net out in terms of value, **but <u>not in terms of cash flow</u>**. Futures are marked to market whereas forwards are not. **This means that the cash and carry trade typically involves negative cash flows throughout the life of the contract, plus a large positive cash flow at maturity.** The profit comes from the fact that the positive cash flow is larger than all the negative flows added up, but the fact remains that the timing is inconvenient.

In order to get the positive cash flow, we have to hold the position for three months, and that means surviving a series of negative cash flows. Not only that, but these negative cash flows <u>might all come at once</u>. Thus the volatility of the spot price of the underlying bill creates liquidity risk for the cash and carry trade.

The fact that markets typically violate the expectations hypothesis is well-known, even if not well-understood. The profitability of the cash and carry trade is not well-known, but perhaps we can understand it as a reward for bearing liquidity risk. What I suggest is that these two anomalies have the same origin, namely a mismatch in the natural forward interest rate positions emerging from the real economy.[2]

[2] A former TA in this course, Daniel Neilson, has done important work showing how deviations between forward and futures rates serve as an observable proxy for unobservable deviations between forward rates and expected spot rates. This is a kind of test of the liquidity premium theory of the term structure.